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CALCULATION OF HEAT TRANSFER ACCOMPANYING FLOW IN PIPES TAKING INTO ACCOUNT THE THERMAL RESISTANCE OF THE WALL

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The starting nonlinear problem is reduced to a simpler form with the help of a linear approximation of the equation relating the tempeature of the outer and inner surfaces of the pipe.

Particular solutions of the problem of convective-radiative heating (cooling) of a liquid in laminar flow in pipes were obtained in [1] by the finite-different method taking into account the transverse thermal resistance of the walls. The mathematical formulation includes the energy equation

$$(1-R^2)\frac{\partial\Theta}{\partial X}=\frac{\partial^2\Theta}{dR^2}+\frac{1}{R}\frac{\partial\Theta}{\partial R} \quad (1)$$

with the boundary conditions:

$$\frac{\partial\Theta}{\partial R}=\mathrm{Bi}\,[1-\vartheta+p(1-\vartheta^4)] \quad \text{at} \quad R=1, \quad (2)$$

$$\frac{\partial\Theta}{\partial R}=0 \quad \text{at} \quad R=0, \quad (3)$$

$$\Theta=\Theta_0 \quad \text{at} \quad X=0 \quad (4)$$

and a relation between the temperature of the outer $\vartheta(X, 1+\Delta)$ and inner $\Theta(X, 1)$ surfaces of the pipe:

$$\vartheta-\beta\mathrm{Bi}\,[1-\vartheta+p(1-\vartheta^4)]=\Theta. \quad (5)$$

Here

$$R=\frac{r}{r_0};\ X=\frac{2x}{\mathrm{Pe}\,d_0};\ \mathrm{Pe}=\frac{W_0 d_0}{a};\ d_0=2r_0;\quad \Theta=\frac{T}{T_c};\ \Theta_0=\frac{T_0}{T_c};$$

$$\mathrm{Bi}=\frac{\alpha r_0}{\lambda}\frac{d}{d_0};\quad p=\frac{\mathrm{Sk}}{\mathrm{Bi}};\ \mathrm{Sk}=\frac{\sigma_w T_c^3 r_0}{\lambda}\frac{d}{d_0};\quad \beta=\frac{\lambda}{\lambda_w}\ln\frac{d}{d_0}.$$

The comparatively large number of parameters makes it difficult to generalize the results of the numerical integration of the system equations (1)-(5). However, the problem (1)-(5) can be simplified. In many cases a function of the type (5) can be approximated with a high degree of accuracy by a linear dependence

$$\vartheta=c+(1-c)\,\Theta, \quad (6)$$

where the constant c is calculated from the relation

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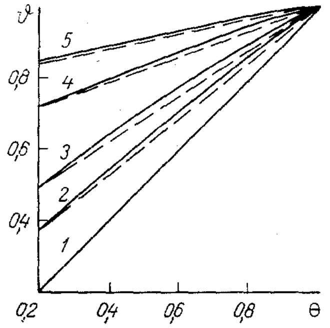

Fig. 1. Graphical representation of the dependence (5): 1) $\beta = 0$; 2) 0.1; 3) 0.2; 4) 0.5; 5) 1.0.

$$c = \frac{\vartheta_0 - \Theta_0}{1 - \Theta_0}, \tag{7}$$

and ϑ_0 is determined from the condition

$$\vartheta_0 - \beta \mathrm{Bi}\,[1 - \vartheta_0 + p\,(1 - \vartheta_0^4)] = \Theta_0. \tag{8}$$

Using Eq. (6) the problem (1)-(5) is reduced to the simpler form

$$(1 - R^2)\frac{\partial U}{dX} = \frac{\partial^2 U}{\partial R^2} + \frac{1}{R}\frac{\partial U}{\partial R}, \tag{9}$$

$$\frac{\partial U}{\partial R} = \mathrm{Bi}^*\,[1 - U + p\,(1 - U^4)] \quad \text{at} \quad R = 1, \tag{10}$$

$$\frac{\partial U}{\partial R} = 0 \quad \text{at} \quad R = 0, \tag{11}$$

$$U = U_0 \quad \text{at} \quad X = 0, \tag{12}$$

where

$$U(X, R) = c + (1 - c)\,\Theta(X, R); \tag{13}$$
$$\mathrm{Bi}^* = (1 - c)\,\mathrm{Bi}; \; U_0 = c + (1 - c)\,\Theta_0.$$

The solutions of the system of Eqs. (9)-(12), obtained by analytical and numerical methods, are presented in [2]. Using these data it is easy to find with the help of Eqs. (13) and (5) the temperature distribution sought in the fluid flow and in the pipe wall.

Figure 1 shows graphs of the surface temperature $\vartheta(X, 1 + \Delta)$ as a function of $\Theta(X, 1)$ for different values of the parameter β. The graphs were constructed using Eq. (5) for $\mathrm{Bi} = 1$ and $p = 1$ (i.e., in this case the radiant component of the total heat flux at the surface of the pipe is greater than the convective component). It follows from the figure that the approximation (6) (broken lines) describes well the behavior of the actual curves (solid lines). It is evident that as p decreases this approximation will be even better, and in the limit $p = 0$ the exact solution is obtained.

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